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Damage assessment of structures with uncertainty by using mode-shape curvatures and fuzzy logic

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ABSTRACT

Uncertainties associated with the structural model and measured vibration data may lead to unreliable damage detection. In this paper, we show that geometric and measurement uncertainty cause considerable problem in damage assessment which can be alleviated by using a fuzzy logic-based approach for damage detection. Curvature damage factor (CDF) of a tapered cantilever beam are used as damage indicators. Monte Carlo simulation (MCS) is used to study the changes in the damage indicator due to uncertainty in the geometric properties of the beam. Variation in these CDF measures due to randomness in structural parameter, further contaminated with measurement noise, are used for developing and testing a fuzzy logic system (FLS). Results show that the method correctly identifies both single and multiple damages in the structure. For example, the FLS detects damage with an average accuracy of about 95 percent in a beam having geometric uncertainty of 1 percent COV and measurement noise of 10 percent in single damage scenario. For multiple damage case, the FLS identifies damages in the beam with an average accuracy of about 94 percent in the presence of above mentioned uncertainties. The paper brings together the disparate areas of probabilistic analysis and fuzzy logic to address uncertainty in structural damage detection.

1. Introduction

Since structural failure can lead to severe economic loss and loss of life, structural damage detection is of great interest to many researchers. Modal-based methods are quite popular for damage detection due to their ease of practical implementation [1,2]. A comprehensive review on modal-based methods is given by Montalvao et al. [1]. Modal methods are based on the fact that the modal parameters (natural frequency, mode shape and modal damping) are functions of the physical parameters (mass, stiffness and damping) and hence it is reasonable to assume that the existence of damage leads to changes in the modal properties of the structure [1]. Damage detection procedures frequently inspect the change in some parameter (e.g. curvature) that takes place when the structure is damaged. Typically, the "footprint", or baseline data set used for comparison can be obtained either from data measured from the undamaged structure, or from a finite element (FE) model of the undamaged structure [3]. The modal parameters used to detect structural damage include frequency response functions (FRF), natural frequencies, mode shapes, mode shape curvatures (MSC), modal flexibility, modal strain energy, etc. [4].

Among these modal parameters, natural frequency is widely used as it can be measured most conveniently and accurately [5]. However, the use of natural frequencies for damage detection is limited for following reasons. First,

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significant damage may cause very small changes in natural frequencies, particularly for larger structures, and these changes may go undetected due to measurement or processing errors. Moreover, variations in the mass of the structure or measurement temperatures may introduce uncertainties in the measured frequency changes [6].

These difficulties with frequency damage indicators can be overcome to some extent by using changes in the mode shapes of any given structure, which are more sensitive to local damage [7–10]. However, using mode shapes also has some drawbacks. First, damage is a local phenomenon and may not significantly influence mode shapes of the lower modes that are usually measured from vibration tests of large structures. Second, extracted mode shapes are affected by environmental noise from ambient loads or inconsistent sensor positions. Third, the number of sensors and the choice of sensor coordinates may have a crucial effect on the accuracy of the damage detection procedure [6].

Given the shortcomings of the frequency and mode shape damage indicators, some researchers used modal curvature (MC) which is the second spatial derivative of the mode shape. Pandey et al. [11] concluded that the absolute changes in the curvature mode shapes are localized in the region of damage and hence can be used to detect damage in a structure. The changes in the curvature mode shapes increase with increasing size of damage.

Sahin and Shenoi [12] presented a damage detection algorithm using a combination of global (changes in natural frequencies) and local (curvature mode shapes) vibration-based analysis data as input in artificial neural networks (ANNs) for location and severity prediction of damage in beam-like structures. Ratcliffe and Bagaria [13] applied gapped smoothing damage detection method to the modal curvature yielding a damage index, which was used for locating a delamination in a composite beam. Hamey et al. [14] concluded about superiority of modal curvature-based methods over other methods by evaluating various damage detection techniques in carbon/epoxy composite beams with several possible damage configurations. Qiao et al. [15] carried out a study to evaluate dynamics-based damage detection techniques (i.e. simplified gapped smoothing method, GSM; generalized fractal dimension, GFD and strain energy method, SEM) for composite laminated plates using smart piezoelectric materials and modern instrumentation like scanning laser vibrometer (SLV). They found that curvature-based methods performed very well for damage detection in composite plates.

Mode shape curvatures are however less reliable when employed for damage detection in structures with multiple damages. All modes should be carefully examined in order to locate all existing faults. Some researchers have conducted studies specifying the limitations of mode shape curvatures in damage detection. Curvature damage factor (CDF) is used for such situations, in which the difference in modal curvature is averaged over all modes. Their studies demonstrated that curvature damage factor accurately localizes the existing damage in a multiple damage scenario. Wahab and Roeck [16] pointed out that the modal curvatures of the lower modes are in general more accurate than those of the higher ones. An extensive measurement grid was required in order to get a good estimation for the higher MC. When more than one fault existed in the structure, it was not possible to locate damage in all positions from the results of only one mode. They proposed curvature damage factor and concluded that when the structure contained multiple damages, the CDF gave a clear identification of these locations. Datta and Talukdar [17] also conclude in their study that CDF gave a clear identification of damage locations, when the structure had multiple damages. Some researchers have proposed combined approaches, in which more than one type of damage signature are used for damage localization. Guo and Zhang [18] proposed a weighted balance evidence theory for localization of multiple damages in structures. They used changes in frequencies and mode shapes as two different information sources, and local decisions were obtained by using the multiple damage location assurance criterion (MDLAC) method and the frequency change damage detection method (FCDDM). They employed evidence fusion theory to integrate these local decisions to make global decisions.

Although the studies on damage detection clearly show the capability of modal curvatures and curvature damage factor, they largely ignored uncertainty issues in damage detection. Since uncertainty in the mode shape gets amplified due to the two differentiation operators required to get the curvature, the solution of inverse problems using this method needs more research.

In general, there are two types of uncertainties in structural dynamics. The first is aleatory or random uncertainty, which occurs due to inherent variabilities or randomness in the system like uncertainty in material properties, geometric properties, etc. The knowledge of experts cannot be expected to reduce aleatory uncertainty although their knowledge may be useful in quantifying the uncertainty. This type of uncertainty is sometimes referred to as irreducible uncertainty. The second type of uncertainty is called epistemic uncertainty and is also referred to as reducible uncertainty and covers unmodeled physics [19]. It is possible to address epistemic uncertainty by sophisticated modeling and most research has concentrated on alleviating epistemic uncertainty. However, aleatory uncertainty is particularly important in damage detection but has been largely ignored.

Uncertainties associated with the mathematical characterization of a structure can lead to unreliable damage detection [20]. Probabilistic analysis provides a tool for incorporating structural uncertainties in the analysis of frequency response by generally describing the uncertainties as random variables. Collins et al. [21] addressed uncertainty in the structural model by treating the initial structural parameters as normally distributed random variables. Random factor method (RFM) and interval factor method (IFM) were proposed by Gao [22] for the analysis of natural frequencies and mode shapes of a truss structure with uncertain physical parameters and geometry. Conventional Monte Carlo simulation (MCS) is the most common and traditional method for a probabilistic analysis [23–25]. Hofer et al. [26] presented an MCS for addressing aleatory uncertainty due to unpredictable variation in the system under study.

Some researchers have studied the effect of uncertainty in damage detection using vibration-based methods. Liu [27] formulated an identification problem by minimizing the error-norm of the eigenequation and addressed the effect of measurement noise on damage detection. Fuzzy logic and continuum damage mechanics were used by Sawyer and Rao [28] to process and analyze the uncertainties and complexities of damaged structure considering measurement noise. Pawar and Ganguli [29] proposed a genetic fuzzy system for damage detection using natural frequencies in helicopter rotor blades having uncertainties in measurement. Chandrashekhar and Ganguli [30] proposed a fuzzy logic system for damage detection in structures having uncertainty in material property as well as measurements. However, these works involving fuzzy logic were restricted to frequencies and did not use modal curvature as the damage indicator.

Selected research has addressed both measurement and model uncertainty. Yong and Hong [31] investigated the impact of uncertainties both in measurements and in the FE model on damage detection using perturbation method. Bakhary et al. [32] have also illustrated damage detection in structures using artificial neural networks considering uncertainties both in the FE model and measured vibration data. Xia [20] proposed that the inaccuracy due to modeling and measurement error can be overcome by taking into account the uncertainties through statistical methods. Again, these studies considered frequency-based damage detection techniques.

When both the Young's modulus and structural geometry are random, it is likely that randomness in geometry will dominate randomness in Young's modulus [33]. Hence, there is a need to address the effect of geometric uncertainties in FE modeling as well as uncertainty due to measurement noise on modal curvature-based damage detection algorithms.

A damage detection problem is an inverse problem, when a damage is estimated by evaluating changes in the structure's response in the damaged state from undamaged state. Neural networks have been very popular for this purpose. However, neural networks have the reputation of being black boxes that are difficult to understand and can also require enormous computer time when the back-propagation algorithm is used. In contrast, fuzzy systems allow for easier understanding as they are expressed in terms of linguistic variables [34]. Fuzzy systems have a built-in fuzzification process at the front end that accounts for uncertainty and does not need to be trained on several cycles of noisy data like neural networks to account for uncertainty [35].

Several other non-destructive techniques (acoustic emission, dye penetrant, stereo X-ray radiography, ultrasonics) with different sensitivity levels are also used for damage detection in structures. Advantages and disadvantages of different available techniques depend on the type of damage to be detected and on the test conditions in which sophisticated laboratory techniques can give highly accurate results. For example, Penetrant-enhanced X-radiography, which utilizes a radio-opaque liquid to infiltrate the examined area, can be used to detect matrix cracks and delaminations. The main drawback of this technique is that it can resolve only damage connected to the surface, while internal defects impossible to fill with the dye may remain undetected [36]. Ultrasonic techniques rely on the use of high-frequency mechanical oscillations for damage detection by evaluating the signal amplitude and/or the time-of-flight of the ultrasonic signal. On the contrary, matrix cracks, lying perpendicularly to the surface, and fiber fracture paths are difficult to detect because they do not offer the wide enough reflecting surface which delaminations present [36]. Due to the complex features of damage mechanisms, more than one method is usually required for a complete non-destructive evaluation of induced damages. The modal-based methods are global in nature and can be used as a complement to local non-destructive techniques.

In this paper, we present a fuzzy logic system (FLS) for damage detection using curvature damage factor. Both single and multiply damaged structures with uncertainties in structural geometric properties as well as measurement noise are considered. Monte Carlo simulation is used for calculating the variation in the structure's curvature damage factor because of uncertainty in the structural parameter. The results of the MCS are used to develop the fuzzy logic system.

2. Modeling of beam

An Euler–Bernoulli beam structure is considered for damage detection in this study. The governing equation of motion for free vibration analysis is given by

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 w(x,t)}{\partial x^2}\right] + m(x)\frac{\partial^2 w(x,t)}{\partial t^2} = 0 \tag{1}$$

where $EI(x)$ is the flexural rigidity of the beam, $m(x)$ is mass per unit length of the beam and $w(x,t)$ is the transverse displacement of the beam reference axis. The beam equation is solved for natural frequencies using the finite element method [37]. The beam is discretized into a number of beam finite elements, with transverse displacement and slope as nodal degrees of freedom and cubic interpolation functions.

For an n degree of freedom system, the equation of motion in discrete form is obtained after assembly of the element matrices and application of the boundary conditions.

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = 0 \tag{2}$$

Here $\mathbf{M}$ is the $n \times n$ mass matrix of the system, $\mathbf{K}$ is the $n \times n$ stiffness matrix of the system, $\mathbf{q}$ is the $n \times 1$ vector of nodal displacements. We seek a solution of the form $\mathbf{q} = \mathbf{\Phi}\,\mathrm{e}^{(\mathrm{i}\omega t)}$, which results in the eigenvalue problem:

$$\mathbf{K}\Phi = \omega^2\mathbf{M}\Phi \tag{3}$$

Solving this eigenvalue problem we get n eigenvalues (ω) and n eigenvectors (Φ) which represent the natural frequencies and natural mode shapes of the system, respectively.

3. Modeling of damage

Damages like crack or delamination result in the reduction of stiffness of a structure locally. Damage in an element is modeled with a damage parameter representing reduction in flexural rigidity of the element. The damage parameter in percentage, D is defined by [35]

$$D = \frac{EI^{(u)} - EI^{(d)}}{EI^{(u)}} 100 \tag{4}$$

where E is Young's modulus of the beam material, I is area moment of inertia of the beam element and the superscripts u and d represent the undamaged and damaged states, respectively.

Two damage conditions are considered. The first case is when the beam has single damage. In this case, the beam is divided into five segments of equal lengths, labeled as "Root", "Inboard", "Center", "Outboard" and "Tip" as shown in Fig. 1. Hence "Root" ranges from 0 to 20 percent of the blade, "Inboard" from 20 to 40 percent, "Center" from 40 to 60 percent, "Outboard" from 60 to 80 percent and "Tip" from 80 to 100 percent. Damage is individually considered in these locations.

The second case is when the beam has multiple damages. Since all possible combinations of multiple damages is very large, a sample of cases is selected to study multiple damages, as has been done by other researchers [16,17]. The beam is divided into 10 segments of equal lengths as shown in Fig. 2. Damage is implanted into two locations along the beam. The first damage location referred as "$\text{Root}_{\text{multiple}}$" consists of first and sixth segments from the fixed end spanning 0–10

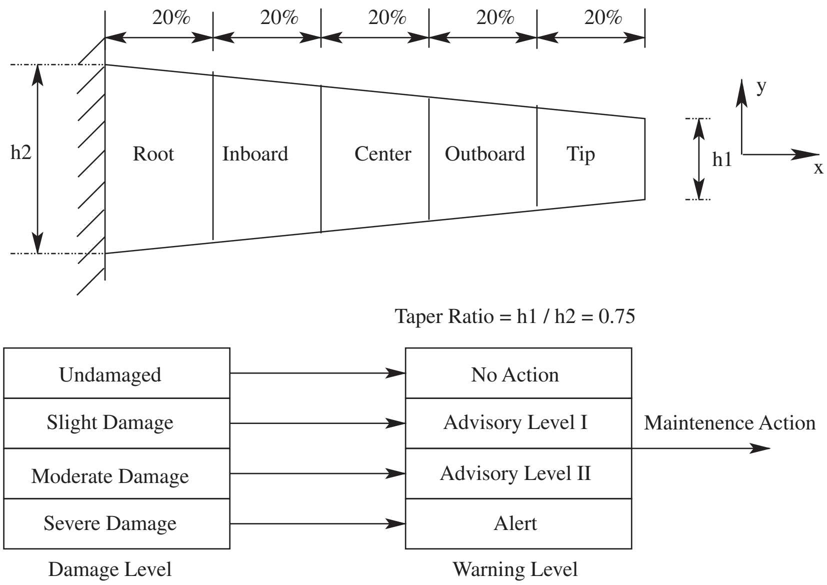

Fig. 1. Schematic representation of beam structure and damage detection system.

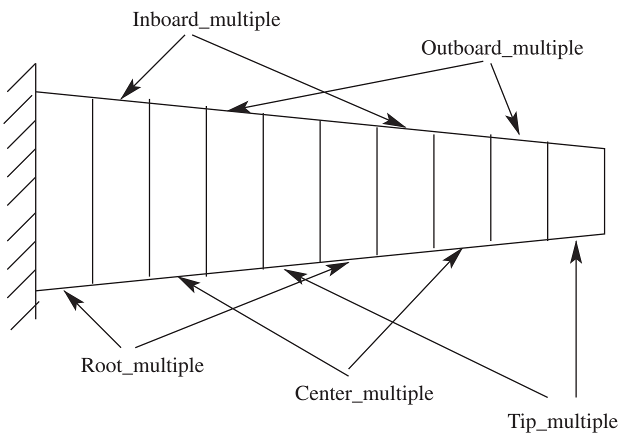

Fig. 2. Schematic representation of the locations of multiple damages in the beam (each segment spans 10 percent of the beam length).

percent and from 50 to 60 percent of the blade. The second damage location "Inboard$_{multiple}$" consists of second and seventh segments spanning 10–20 percent and from 60 to 70 percent of the blade. The third damage location "Center$_{multiple}$" consists of third and eighth segments spanning 20–30 percent and from 70 to 80 percent of the blade. The fourth damage location "Outboard$_{multiple}$" consists of fourth and ninth segments spanning 30–40 percent and from 80 to 90 percent of the blade. The fifth damage location "Tip$_{multiple}$" consists of fifth and tenth segments from the fixed end of the beam spanning 40–50 percent and from 90 to 100 percent of the blade.

The structural damage in each segment is simulated by stiffness reduction (D) of 20 percent, 40 percent and 60 percent. These damages are classified as "Slight Damage", "Moderate Damage" and "Severe Damage", respectively. Damage sizes below "Slight Damage" are classified as "Undamaged" and damage sizes greater than "Severe Damage" are classified as "Catastrophic Damage". This type of representation of structural damage along various locations of the beam helps in development of a user-friendly decision system.

4. Damage indicator

The curvature damage factor is used as damage indicator. CDF is obtained by averaging the difference between the modal curvatures of the damaged and undamaged beam for the first six modes. Reduction in the stiffness at some location for the damaged structure results in an increase of the local modal curvature.

Central difference approximation is used to estimate mode shape curvatures from the mass normalized mode shapes obtained from the finite element analysis. Numerically, it is obtained as [11]

$$\bar{v}_{i,j} = \frac{\phi_{(i+1),j} - 2\phi_{i,j} + \phi_{(i-1),j}}{h^2} \tag{5}$$

where $\bar{v}_{i,j}$ represents modal curvature, first subscript i represents node number, second subscript j represents the corresponding mode number, h represents element length as the beam is discretized with elements of equal length and $\phi_{i,j}$ represents the mass normalized modal value for the ith node in the jth mode.

The change in mode shape curvature (CMSC) is obtained by subtracting the undamaged mode shape curvature vector from the respective damaged MSC, given by

$$\Delta\bar{v}_{i,j} = \bar{v}_{i,j}^{(d)} - \bar{v}_{i,j}^{(u)} \tag{6}$$

The change in mode shape curvature obtained at each node of the beam FE model is normalized to the same range using

$$\Delta v_{i,j} = \left[1 + \frac{\Delta\bar{v}_{i,j}}{\max(\Delta\bar{v}_{i,j}) - \min(\Delta\bar{v}_{i,j})}\right]^2 \tag{7}$$

The curvature damage factor is obtained by averaging these normalized change in mode shape curvatures over all the selected modes. Mathematically, CDF for ith node for first N modes is given by

$$CDF_i = \frac{1}{N}\sum_{j=1}^{N} \Delta v_{i,j} \tag{8}$$

Therefore, different combinations of the three damage levels with five different locations of the beam give different sets of measurement deltas (CDF), which are used to create the knowledge base composed of fuzzy rules. All results in the paper are for the normalized modal curvatures.

5. Uncertainty modeling

There is always some difference between predictions by mathematical models and test results due to associated uncertainties. Modeling uncertainty can result from uncertainties in geometric properties, material property and also due to assumptions made in the modeling. Randomness in geometric property (taper angle of the beam) is considered in this study as a representation of uncertainty in FE modeling. Investigation from literature shows that the randomness in the geometric properties in the thickness direction (y-direction in Fig. 1) for beam type structures is usually taken from 0.0 percent to 2.0 percent [38]. Hence to capture the uncertainty, we take 1.0 percent coefficient of variation (COV) of the thickness (y-direction in Fig. 1) at the free end of the cantilever beam, which in turn results in uncertainty in the beam taper angle. COV is a normalized measure of the dispersion of a probability distribution. COV is defined as the ratio of the standard deviation to the mean for a non-zero mean random process.

In addition to modeling uncertainty, noise may be present in the measured data. This measurement uncertainty can originate from sensor noise and measurement errors. Though the use of modern instruments has reduced measurement uncertainty, it can never be eliminated. It can therefore be expected that uncertainty is present in the measurement deltas (CDF). We shall assume uniformly distributed noise of about 10 percent to be present in the measurement delta [35].

The noisy simulated measurement delta (CDF_{noisy}), obtained after adding measurement noise to the randomized measurement delta (CDF_{random}) is given by [28]

$$CDF_{\text{noisy}} = CDF_{\text{random}}(1 + u\alpha) \tag{9}$$

where CDF_{random} is the randomized measurement delta due to uncertainty in geometric property, u is a random number in the interval $[-1, 1]$ and α is a noise level parameter.

6. Uncertainty quantification

A tapered steel cantilever beam is used for the numerical results. Relevant properties of the beam are shown in Table 1 [29]. First six natural frequencies for the undamaged beam are 147, 820, 2217, 4304, 7088 and 10 569 rad per second, respectively. The beam is divided into 20 finite elements of equal length. Each segment spanning 20 percent of the blade in Fig. 1 is therefore divided into four finite elements for the single damage case. For the multiple damage case in Fig. 2, each segment consists of two elements for the damage at each location.

The Monte Carlo simulation method [31,39,40] and perturbation method [41,42] are widely used for the dynamic characteristics analysis of structures with random parameters. Papadopoulos et al. [43] proposed finite element method-based fast Monte Carlo simulation procedure (FEM-FMCS) for the analysis of the mean and mean square response of stochastic structural systems whose material properties were described by random fields. Stefanou and Papadrakakis [44] used the spectral representation method for the description of the random fields in conjunction with MCS for the computation of the response variability of shell structures. We use MCS in this study as it is a non-intrusive method which is easy to implement.

The variation in structural parameter represents the uncertainty associated with FE modeling and the added noise in the calculated measurement deltas simulates the uncertainty present in the experimental measurement. A large number of measurement deltas (CDF_{random}) are obtained using Monte Carlo simulation on the beam FE model by varying the structural parameter as a random variable with COV of 1 percent. The spread of the measurement deltas are estimated from MCS for 5000 input data points of the random variable.

Figs. 3 and 4 show the deterministic value and the minimum and maximum values of the damage indicator obtained using the MCS with geometric uncertainty. There are large overlaps in the measurement delta CDF's (the damage indicator) for the faults with different damage levels at same location due to uncertainty in the physical parameter itself. The peaks of the measurement CDF are good indicators for damage locations for single damage (Fig. 3) as well as multiple damage (Fig. 4).

If measurement noise is added in addition to geometric uncertainty, the overlaps increase even further as shown in Figs. 5 and 6, reducing the probability of the success of damage identification. In these cases measurement noise of $\alpha = 0.10$ is added. It is obvious from Figs. 3–6 that the CDF clearly identifies the damage locations but it is very difficult to quantify the damage size in the presence of uncertainty. Also, automated reasoning is needed to convert the CDF data (measurement deltas) in graphical form to a message indicating the location and size of damage. A fuzzy logic system is now developed to perform this automated reasoning function under modeling and measurement uncertainty.

7. Fuzzy logic system

Fuzzy set theory is a marvelous tool for modeling the kind of uncertainty associated with vagueness, with imprecision and/or with a lack of information regarding a particular element of problem at hand [45]. A typical fuzzy logic system uses four basic components: rules, fuzzifier, inference engine and defuzzifier (Fig. 7) to nonlinearly map an input feature vector into a scalar output. The development of the FLS is briefly described in this section.

7.1. Input and output

Inputs to the FLS are measurement deltas and outputs are structural damage size and location. We have a measurement vector (CDF_i) represented by **x** and five damage locations represented by **y**. The objective is to find a functional mapping between **x** and **y**. Mathematically, this can be represented as

$$\mathbf{y} = \mathbf{F}(\mathbf{x}) \tag{10}$$

Table 1
Material and geometric properties of the beam.

Young's modulus (E)	$= 2.0 \times 10^5$ N/mm^2
Width of the beam	$= 24$ mm
Height of the beam at fixed end (h_1)	$= 10$ mm
Height of the beam at free end (h_2)	$= 7.5$ mm
Taper ratio (h_2/h_1)	$= 0.75$
Length	$= 600$ mm
Mass density	$= 7840 \times 10^{-9}$ kg/mm^3

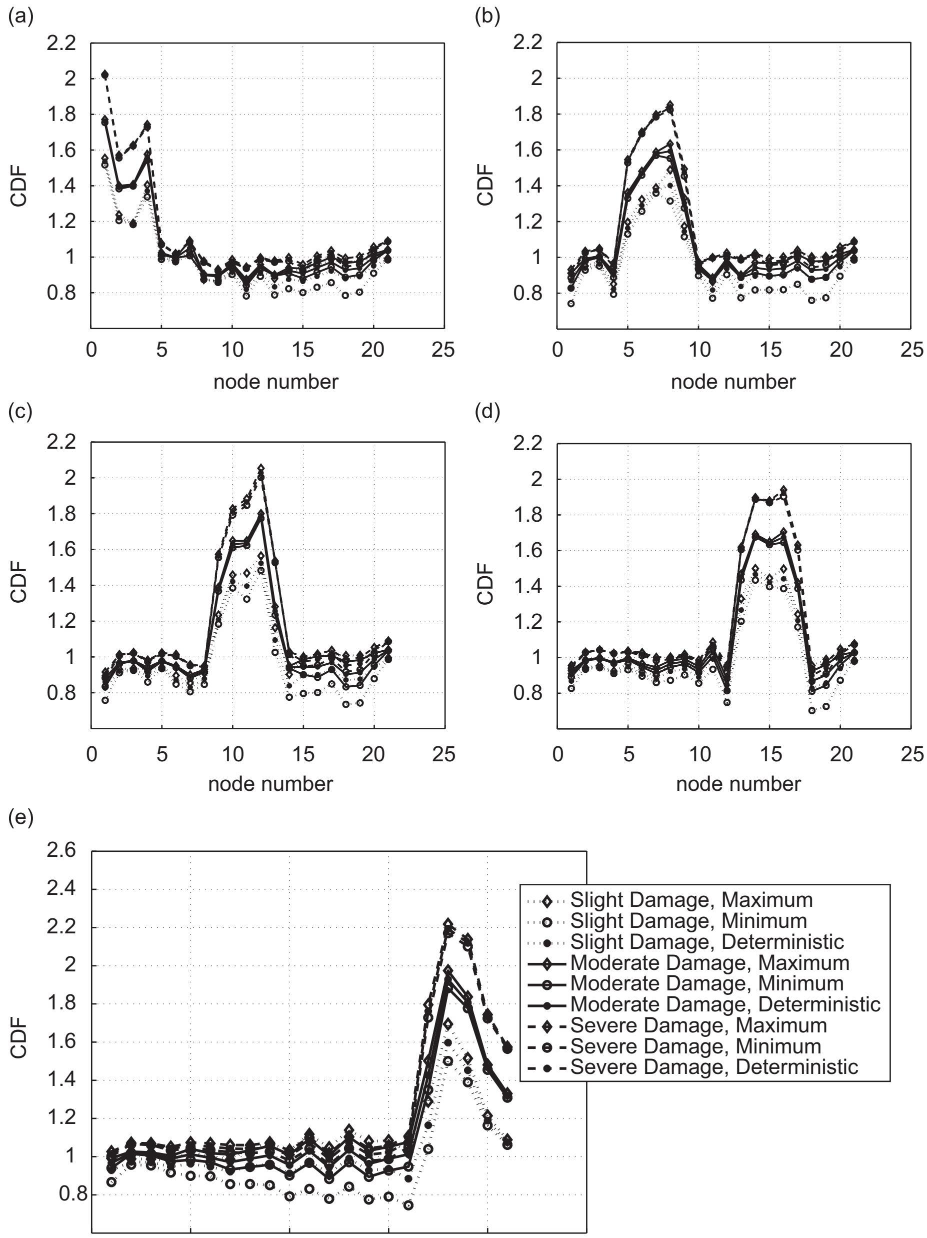

Fig. 3. Variability of the curvature damage factor at various damage levels for single damage cases with geometric uncertainty only: (a) Damage at Root; (b) Damage at Inboard; (c) Damage at Center; (d) Damage at Outboard and (e) Damage at Tip.

where $\mathbf{y} = \{\text{Root, Inboard, Center, Outboard, Tip}\}^{\mathrm{T}}$ for single damage, or $\mathbf{y} = \{\text{Root}_{multiple}, \text{Inboard}_{multiple}, \text{Center}_{multiple}, \text{Outboard}_{multiple}, \text{Tip}_{multiple}\}^{T}$ for multiple damage, and $\mathbf{x} = \{CDF_i\}^{\mathrm{T}}$, where i varies from 1 to 21. Thus, CDF_i represents the noisy CDF vector corresponding to the given structural fault.

7.2. Fuzzification

We consider two different damage conditions. First, when the structure has single damage and second, when the structure has multiple damage locations. The structural damages are considered as crisp numbers.

To get a degree of resolution of the extent of damage, each of these damage locations is allowed several levels of damage and split into linguistic variables. For example, consider "root" as a linguistic variable. It can be decomposed into a set of

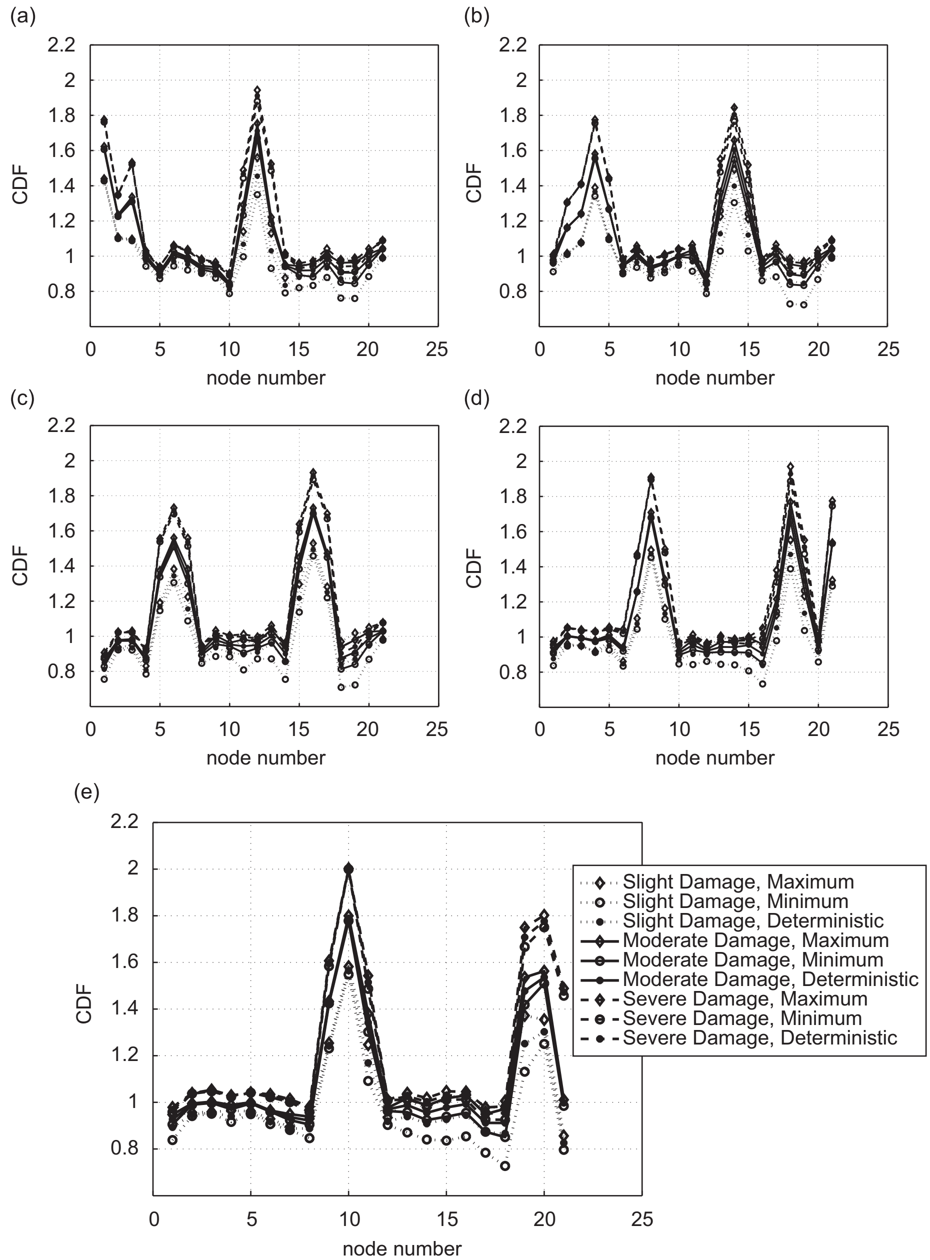

Fig. 4. Variability of the curvature damage factor at various damage levels for multiple damage cases with geometric uncertainty only: (a) Damage at $\text{Root}_{\text{multiple}}$; (b) Damage at $\text{Inboard}_{\text{multiple}}$; (c) Damage at $\text{Center}_{\text{multiple}}$; (d) Damage at $\text{Outboard}_{\text{multiple}}$ and (e) Damage at $\text{Tip}_{\text{multiple}}$.

terms: $T(\text{root}) = (\text{Undamaged, Slight Damage, Moderate Damage, Severe Damage, Catastrophic Damage})$, where each term in $T(\text{root})$ is characterized by a fuzzy set in the universe of discourse $U(\text{root}) = \{0, 70\}$ corresponding to the percent reduction in stiffness used to model damage. The other structural damage variables are fuzzified in a similar manner.

The measurement deltas CDF_i are also treated as fuzzy variables. To get a high degree of resolution, they are further split into linguistic variables. For example, consider CDF_i as a linguistic variable. It can be decomposed into a set of terms: $T(CDF_i) = (\text{Negligible, Very Low, Low, Low Medium, Medium, Medium High, High, Very High})$, where each term in $T(CDF_i)$ is characterized by a fuzzy set in the universe of discourse $U(CDF_i) = \{0.5, 2.5\}$. Measurement deltas larger than covered by the universe of discourse will represent an extensive structural damage indicative of a catastrophic failure and are not considered. We want to detect the damage much before it becomes catastrophic.

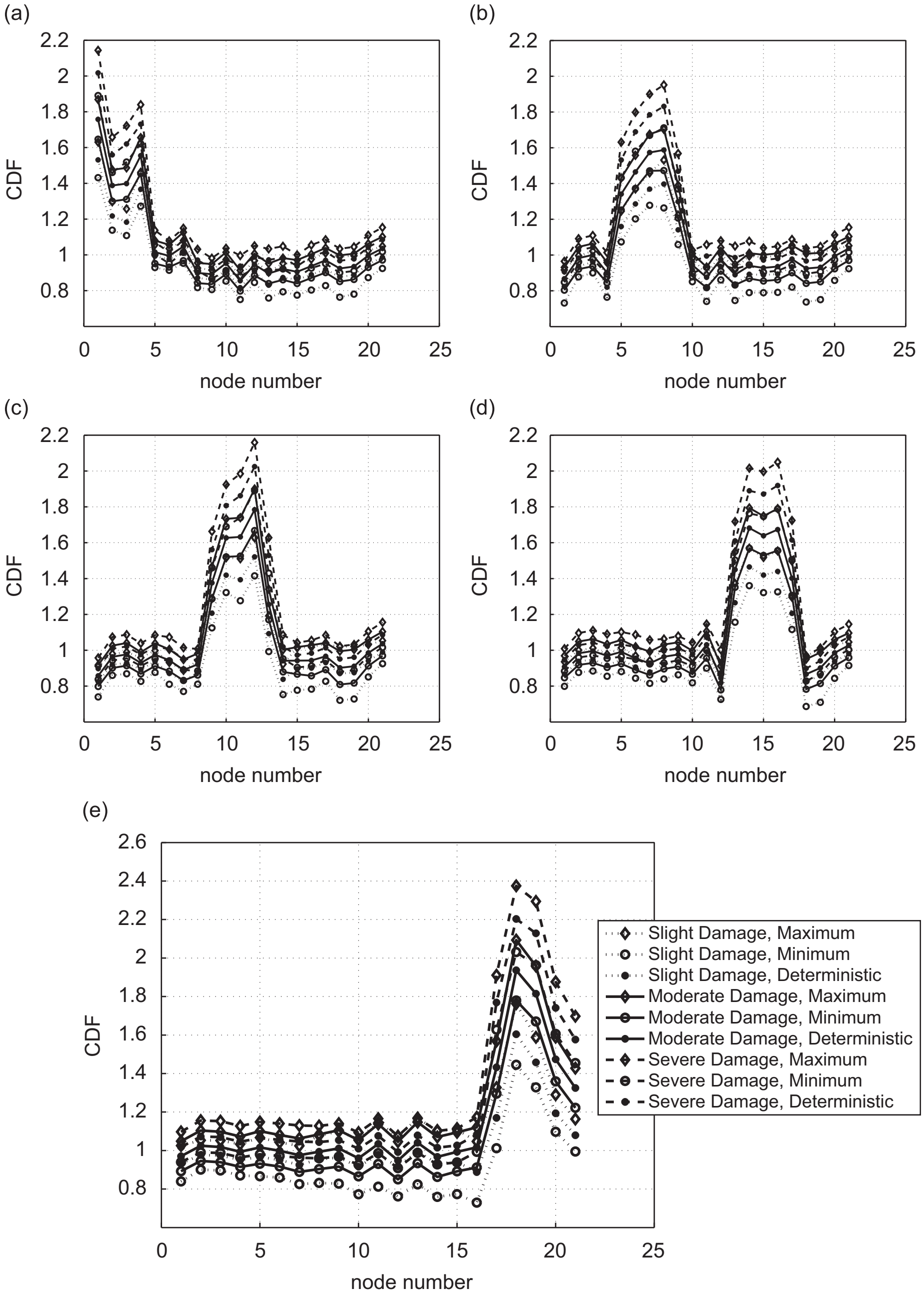

Fig. 5. Variability of the curvature damage factor at various damage levels for single damage cases with geometric uncertainty and measurement noise: (a) Damage at Root; (b) Damage at Inboard; (c) Damage at Center; (d) Damage at Outboard and (e) Damage at Tip.

Fuzzy sets with Gaussian membership functions are used for the input variables. These fuzzy sets can be defined using the following equation:

$$\mu(x) = e^{-0.5((x-m)/\sigma)^2} \tag{11}$$

where m is the midpoint of the fuzzy set and σ is the spread (standard deviation) associated with the variable. Gaussian fuzzy membership functions are quite popular in the fuzzy logic literature, as they are the basis for the connection between fuzzy system and radial basis functions (RBF) neural networks [46]. Table 2 gives the linguistic measure associated with each fuzzy set and the midpoint of the set for each measurement delta. The midpoints of the fuzzy sets are selected to capture the variability of the damage indicator (CDF_i) due to uncertainty at different damage conditions. We select different

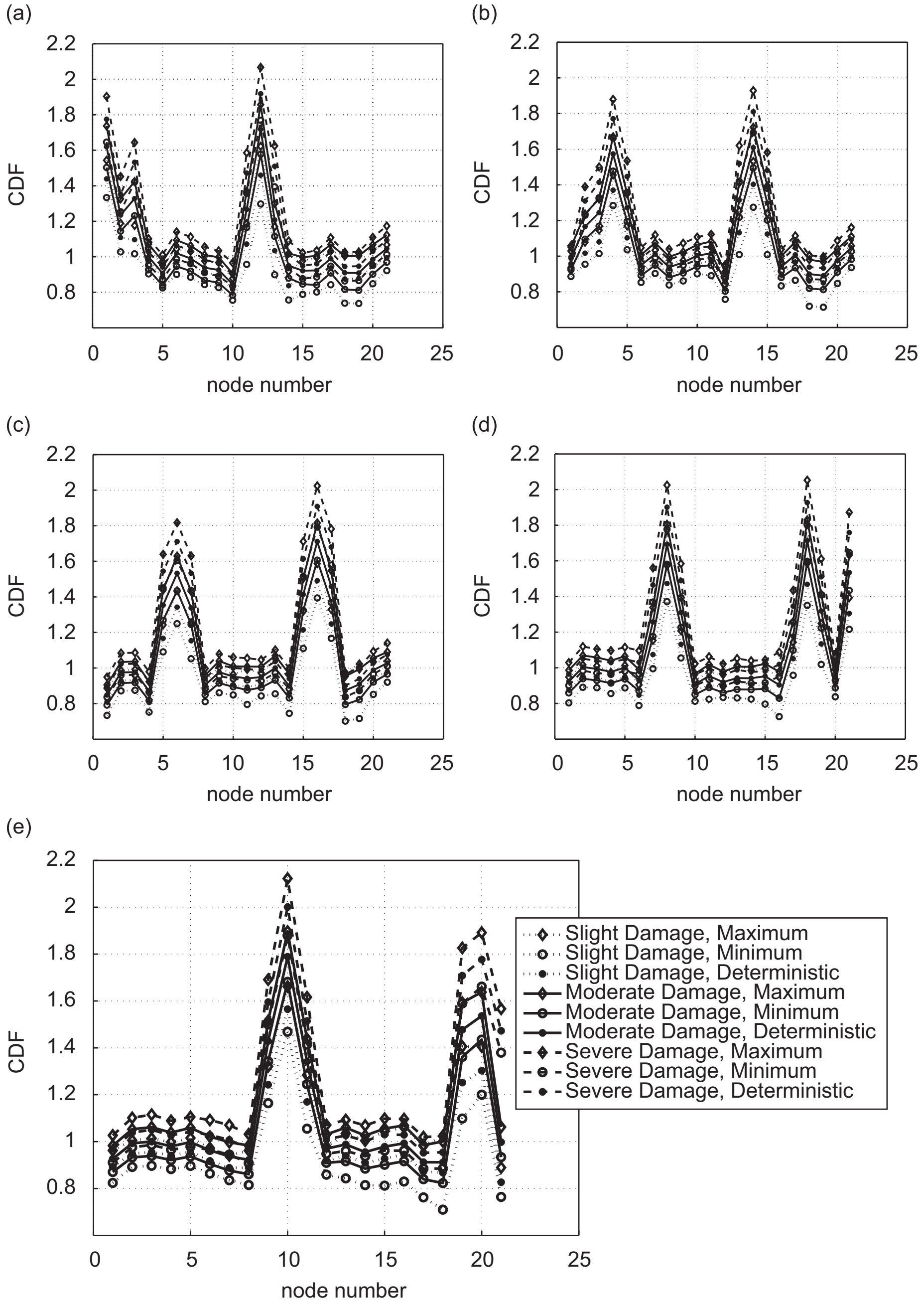

Fig. 6. Variability of the curvature damage factor at various damage levels for multiple damage cases with geometric uncertainty and measurement noise: (a) Damage at $\text{Root}_{\text{multiple}}$; (b) Damage at $\text{Inboard}_{\text{multiple}}$; (c) Damage at $\text{Center}_{\text{multiple}}$; (d) Damage at $\text{Outboard}_{\text{multiple}}$ and (e) Damage at $\text{Tip}_{\text{multiple}}$.

spacings between two successive fuzzy sets. This maximizes the probability of capturing the variability of a measurement delta in any one of the fuzzy sets for each nodal point for a damage case. A proper tuning of the membership functions is required as each time the mean and standard deviation of the damage indicator for a node change for every different fault (Figs. 3 and 4). Even different damage levels at a location force different membership functions to be selected for the fuzzy sets. Hence a compromise is made in selecting the membership functions to maximize the overall success rate of the FLS while maintaining uniqueness of fuzzy rules.

The effective width of a fuzzy set (A) can be typically given by half of the difference between the midpoints of the adjacent fuzzy set to the right of A and the adjacent fuzzy set to the left of A. This is true if all the fuzzy sets are defined by the membership functions having same standard deviation. We require different effective width of fuzzy sets at different

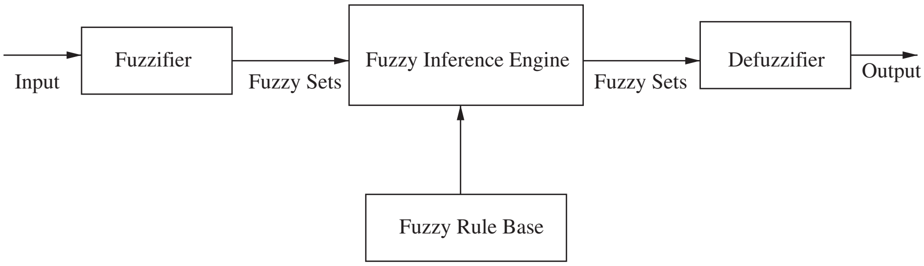

Fig. 7. Schematic of a typical fuzzy logic system.

Table 2
Gaussian fuzzy sets.

Linguistic measure	Symbol	Midpoint CDF
Negligible	N	0.70
Very Low	VL	1.00
Low	L	1.25
Low–Medium	LM	1.50
Medium	M	1.70
Medium–High	MH	1.90
High	H	2.10
Very High	VH	2.30

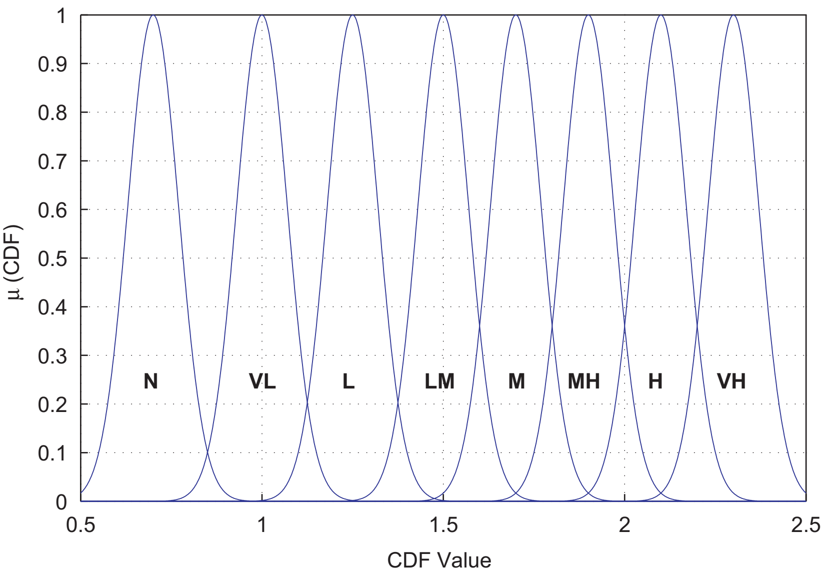

Fig. 8. Fuzzy sets representing measurement deltas over universe of discourse (0.5–2.5).

CDF levels. Figs. 3 and 4 show that lower CDF levels require higher effective width and higher CDF levels require lower effective width to capture the variability and to maintain the uniqueness. Hence we select variable spacing between the midpoints of membership functions for the fuzzy sets at different CDF levels (Fig. 8). The information of variability of the damage indicator obtained by the probabilistic simulation is thus exploited to develop the fuzzy logic system.

Fig. 8 shows the membership functions for each of the eight input fuzzy sets. The standard deviations for the membership functions are selected as 0.07 to provide enough intersection and symmetry between the fuzzy sets so as to give good accuracy of detection [35]. Some level of intersection between fuzzy sets is needed to address the uncertainty of measurement inputs in the FLS. Though probability and fuzzy logic have been seen as disparate topics in handling uncertainty, we can link them through the use of Gaussian fuzzy sets as they are based on normal distribution functions.

Table 3
Rules for fuzzy system for "Undamaged" and the faults at "Root" and "Inboard" for Single damage case.

Node no.	Measurement deltas (CDF)						
	Undamaged	Damage at Root			Damage at Inboard		
	Undamaged	Slight Damage	Moderate Damage	Severe Damage	Slight Damage	Moderate Damage	Severe Damage
1	VL	LM	MH	H	N	VL	VL
2	VL	L	LM	LM	VL	VL	VL
3	VL	L	LM	M	VL	VL	VL
4	VL	LM	M	M	N	VL	VL
5	VL	VL	VL	VL	L	L	LM
6	VL	VL	VL	VL	L	LM	M
7	VL	VL	VL	VL	L	LM	M
8	VL	VL	VL	VL	L	M	M
9	VL	VL	VL	VL	L	L	LM
10	VL	VL	VL	VL	VL	VL	VL
11	VL	N	VL	VL	N	VL	VL
12	VL	VL	VL	VL	VL	VL	VL
13	VL	N	VL	VL	N	VL	VL
14	VL	VL	VL	VL	VL	VL	VL
15	VL	VL	VL	VL	VL	VL	VL
16	VL	VL	VL	VL	VL	VL	VL
17	VL	VL	VL	VL	VL	VL	VL
18	VL	VL	VL	VL	N	VL	VL
19	VL	VL	VL	VL	VL	VL	VL
20	VL	VL	VL	VL	VL	VL	VL
21	VL	VL	VL	VL	VL	VL	VL

7.3. Rule generation

Rules for the fuzzy system are obtained by fuzzification of the numerical values obtained from the finite element analysis using the following procedure [35]:

1. The CDF vector corresponding to a given structural fault is input to the FLS and the degree of membership of the elements of CDF are obtained. Therefore, each measurement delta has eight degree of memberships based on the linguistic measures in Table 2.
2. Each measurement delta is then assigned to the fuzzy set with maximum degree of membership.
3. One rule is obtained for each fault by relating the measurement deltas with maximum degree of membership to a fault.

The fuzzy rules are given in Tables 3–6. The linguistic symbols used in these tables are defined in Table 2. There is a separate fuzzy rule set for each fault location which can be interpreted as a pattern classifier.

The rule for the "Undamaged case" is given in Table 3 and shows "Very Low" level of change in all the measurements. Rules for the fault "Damage at root" at different damage severity level for the CDF vector are also given in Table 3. Here we clearly see the different levels of changes for the various measurements. As damage becomes more severe, the indicators move from "Low–Medium" to "Medium–High" and "High" levels. The rules for the other faults given in Tables 3–6 can be similarly interpreted.

A close observation of the tables from Tables 3–6 shows that each rule represents a unique signature and is different from all the other rules. Therefore, the fuzzy system is a good pattern classifier. These rules provide a knowledge base and represent how a human engineer would interpret data to isolate structural damage using changes in CDF's. It is difficult for a human to memorize and process such a large number of patterns. However, the task is straight forward for computer interpretation.

8. Damage assessment

Once the fuzzy rules are applied to a given measurement, we have a set of degree of memberships for each fault. For fault isolation, we are interested in the most likely fault. Therefore, the so-called maximum matching method has been widely used for defuzzification in damage detection work [47,48]. In this paper, we develop a new defuzzification technique called the "sliding window method" using CDF vectors. This method is briefly described below. Details about numerical implementation of the sliding window method using scalar eigenvalues representing frequencies can be found in [30].

Table 4
Rules for fuzzy system for the faults at "Center", "Outboard" and "Tip" for Single damage case.

Node no.	Measurement deltas (CDF)								
	Damage at Center			Damage at Outboard			Damage at Tip		
	Slight Damage	Moderate Damage	Severe Damage	Slight Damage	Moderate Damage	Severe Damage	Slight Damage	Moderate Damage	Severe Damage
1	N	N	VL	VL	VL	VL	VL	VL	VL
2	VL	VL	VL	VL	VL	VL	VL	VL	VL
3	VL	VL	VL	VL	VL	VL	VL	VL	VL
4	VL	VL	VL	VL	VL	VL	VL	VL	VL
5	VL	VL	VL	VL	VL	VL	VL	VL	VL
6	VL	VL	VL	VL	VL	VL	VL	VL	VL
7	N	VL	VL	VL	VL	VL	VL	VL	VL
8	VL	VL	VL	VL	VL	VL	VL	VL	VL
9	L	L	LM	VL	VL	VL	VL	VL	VL
10	LM	LM	MH	VL	VL	VL	VL	VL	VL
11	LM	LM	MH	VL	VL	VL	VL	VL	VL
12	LM	M	H	N	N	VL	VL	VL	VL
13	VL	L	LM	L	LM	LM	VL	VL	VL
14	N	VL	VL	LM	M	MH	VL	VL	VL
15	VL	VL	VL	LM	M	MH	VL	VL	VL
16	VL	VL	VL	LM	M	MH	VL	VL	VL
17	VL	VL	VL	L	LM	LM	L	L	LM
18	VL	VL	VL	N	N	VL	M	MH	MH
19	VL	VL	VL	VL	VL	VL	LM	M	MH
20	VL	VL	VL	VL	VL	VL	L	LM	LM
21	VL	VL	VL	VL	VL	VL	VL	L	LM

Table 5
Rules for fuzzy system for "Undamaged" and the faults at "$\text{Root}_{\text{multiple}}$" and "$\text{Inboard}_{\text{multiple}}$" for multiple damage case.

Node no.	Measurement deltas (CDF)						
	Undamaged	Damage at $\text{Root}_{\text{multiple}}$			Damage at $\text{Inboard}_{\text{multiple}}$		
	Undamaged	Slight Damage	Moderate Damage	Severe Damage	Slight Damage	Moderate Damage	Severe Damage
1	VL	LM	M	M	VL	VL	VL
2	VL	VL	L	L	VL	L	L
3	VL	VL	L	LM	VL	L	L
4	VL	VL	VL	VL	L	M	M
5	VL	VL	VL	VL	VL	L	LM
6	VL	VL	VL	VL	VL	VL	VL
7	VL	VL	VL	VL	VL	VL	VL
8	VL	VL	VL	VL	N	VL	VL
9	VL	VL	VL	VL	VL	VL	VL
10	VL	N	VL	VL	VL	VL	VL
11	VL	VL	L	LM	VL	VL	VL
12	VL	LM	M	MH	N	VL	VL
13	VL	VL	L	LM	VL	L	LM
14	VL	N	VL	VL	L	M	M
15	VL	VL	VL	VL	VL	L	LM
16	VL	VL	VL	VL	N	VL	VL
17	VL	VL	VL	VL	VL	VL	VL
18	VL	N	VL	VL	N	VL	VL
19	VL	N	VL	VL	N	VL	VL
20	VL	VL	VL	VL	VL	VL	VL
21	VL	VL	VL	VL	VL	VL	VL

8.1. Sliding window method

We take a window with maximum and minimum limits as represented by the maximum CDF_{noisy} and the minimum CDF_{noisy} of a set of measurement CDF_{noisy} for any given arbitrary encountered fault. Then this window is slid over the fuzzy rule base sets with the maximum move limit (M) being equal to the difference between the midpoints of two successive fuzzy sets. The maximum move limit for the negative side is $CDF_{\text{noisy}} - M/2$ and the maximum move limit for the positive

Table 6
Rules for fuzzy system for the faults at "Center$_{multiple}$", "Outboard$_{multiple}$" and "Tip$_{multiple}$" for multiple damage case.

Node no.	Measurement deltas (CDF)								
	Damage at Center$_{multiple}$			Damage at Outboard$_{multiple}$			Damage at Tip$_{multiple}$		
	Slight Damage	Moderate Damage	Severe Damage	Slight Damage	Moderate Damage	Severe Damage	Slight Damage	Moderate Damage	Severe Damage
1	N	VL	VL	VL	VL	VL	VL	VL	VL
2	VL	VL	VL	VL	VL	VL	VL	VL	VL
3	VL	VL	VL	VL	VL	VL	VL	VL	VL
4	N	VL	VL	VL	VL	VL	L	VL	VL
5	L	L	LM	VL	VL	VL	VL	VL	VL
6	LM	LM	M	VL	VL	VL	VL	VL	VL
7	L	L	LM	VL	L	LM	VL	VL	VL
8	VL	VL	VL	LM	M	MH	N	VL	VL
9	VL	VL	VL	L	L	LM	VL	LM	LM
10	VL	VL	VL	VL	VL	VL	VL	MH	MH
11	VL	VL	VL	VL	VL	VL	VL	L	LM
12	VL	VL	VL	VL	VL	VL	N	VL	VL
13	VL	VL	VL	VL	VL	VL	VL	VL	VL
14	VL	VL	VL	VL	VL	VL	L	VL	VL
15	L	LM	LM	VL	VL	VL	VL	VL	VL
16	LM	M	MH	N	VL	VL	N	VL	VL
17	L	LM	M	VL	L	L	VL	VL	VL
18	N	VL	VL	LM	M	MH	N	VL	VL
19	VL	VL	VL	L	L	LM	N	LM	M
20	VL	VL	VL	VL	VL	VL	VL	LM	M
21	VL	VL	VL	L	LM	M	VL	VL	LM

side is $CDF_{noisy} + M/2$. At each step, the window is slid over the fuzzy sets (i.e. each of the elements of CDF_{noisy} vector of the encountered fault given a step increment), the FLS identifies a fault with highest degree of membership and each of the different identified faults is counted. For fault isolation, the fault with the maximum count identified by the FLS is selected as the most likely fault.

The FLS is tested for the measurement deltas obtained from FE model considering randomness in the structural parameter further contaminated with measurement noise. In each case, 5000 noisy data points are generated for each seeded fault and the percentage success rate for the fuzzy system in isolating a fault is calculated.

If the uncertainty level is larger than or close to the changes in the damage indicator due to damage, the true information would be submerged in the noise. Then the actual damaged members may not be identified accurately and/or the healthy members may be wrongly detected as damaged resulting in a false warning [32]. However, this being the case, the FLS tested with these noisy data (CDF_{noisy}) having large variation and overlaps gives very good success rate. We will next study the effects of different levels of measurement and geometric uncertainty on the damage assessment capability of the FLS. Note that the FLS was developed with a geometric uncertainty in taper of 1 percent COV with a measurement uncertainty $\alpha = 0.10$.

Tables 7 and 8 show results obtained for fault isolation using both the methods (i.e. (1) with highest degree of membership [35] and (2) with sliding window technique). The sliding window method gives a higher success rate for fault isolation than the FLS [35] based on maximum degree of membership for fault classification. The increase in success rate of about 14 percent is quite significant. Further results are obtained using the sliding window defuzzifier.

8.2. Simulations with uncertain data

We consider several combinations of geometric uncertainty and noise levels to evaluate the robustness of the FLS. The FLS defined by the rules of CDF vector inputs is tested for the noisy measurement deltas with different levels of measurement noise ($\alpha = 0.10$ and 0.15). These CDF vectors also include the effect of geometric uncertainty in the beam taper ratio. Three different levels of geometric uncertainty in beam taper ratio (i.e. 0.5 percent, 1.0 percent and 1.5 percent of COV as taper is a linear function of the beam height) are considered. Table 9 shows the success rate for each rule for different values of α and geometric uncertainty for the single damage case. Table 10 shows the success rate for each rule for different values of α and geometric uncertainty for the multiple damage case. These results are summarized in bar plots shown in Fig. 9 for single damage case and Fig. 10 for multiple damage case. The results show that the FLS is robust to uncertainty.

The FLS tested with different levels of measurement noise gives an average success rate of 94.62 percent for added noise level $\alpha = 0.10$ and 87.81 percent for added noise level $\alpha = 0.15$ when the beam has single damage and geometric uncertainty of 1 percent COV. The FLS classifies multiple damages with similar accuracies. It gives an average success rate

Table 7
Success rate in percent for the FLS with different defuzzification techniques at noise level $(\alpha) = 0.10$ for single damage case with geometric uncertainty of 1.0 percent COV in beam taper angle.

Rule no.	Highest degree of membership [35]	Sliding window
Undamaged	88.42	99.68
Slight Damage at Root	72.63	94.25
Moderate Damage at Root	77.31	93.63
Severe Damage at Root	82.73	97.41
Slight Damage at Inboard	75.92	87.61
Moderate Damage at Inboard	83.53	96.35
Severe Damage at Inboard	86.21	98.32
Slight Damage at Center	79.01	92.37
Moderate Damage at Center	81.46	95.92
Severe Damage at Center	79.52	94.63
Slight Damage at Outboard	71.53	86.31
Moderate Damage at Outboard	87.87	96.58
Severe Damage at Outboard	81.89	98.46
Slight Damage at Tip	81.62	94.20
Moderate Damage at Tip	79.37	93.73
Severe Damage at Tip	82.48	94.52
Average S_R	80.71	94.62

Table 8
Success rate in percent for the FLS with different defuzzification techniques at noise level $(\alpha) = 0.10$ for multiple damage case with geometric uncertainty of 1.0 percent COV in beam taper angle.

Rule no.	Highest degree of membership [35]	Sliding window
Undamaged	88.42	99.68
Slight Damage at $\text{Root}_{\text{multiple}}$	76.24	93.14
Moderate Damage at $\text{Root}_{\text{multiple}}$	81.27	95.43
Severe Damage at $\text{Root}_{\text{multiple}}$	80.53	96.21
Slight Damage at $\text{Inboard}_{\text{multiple}}$	77.85	89.64
Moderate Damage at $\text{Inboard}_{\text{multiple}}$	83.73	98.20
Severe Damage at $\text{Inboard}_{\text{multiple}}$	77.54	92.13
Slight Damage at $\text{Center}_{\text{multiple}}$	79.63	90.47
Moderate Damage at $\text{Center}_{\text{multiple}}$	83.76	97.30
Severe Damage at $\text{Center}_{\text{multiple}}$	81.78	93.76
Slight Damage at $\text{Outboard}_{\text{multiple}}$	76.42	88.35
Moderate Damage at $\text{Outboard}_{\text{multiple}}$	77.81	93.19
Severe Damage at $\text{Outboard}_{\text{multiple}}$	80.72	95.38
Slight Damage at $\text{Tip}_{\text{multiple}}$	79.34	90.61
Moderate Damage at $\text{Tip}_{\text{multiple}}$	75.89	93.27
Severe Damage at $\text{Tip}_{\text{multiple}}$	79.65	93.60
Average S_R	80.03	93.77

of 93.77 percent for added noise level $\alpha = 0.10$ and 86.85 percent for added noise level $\alpha = 0.15$ when the beam has multiple damage and geometric uncertainty of 1 percent COV. It also classifies the undamaged structure with an accuracy of 99.23 percent for added noise level up to $\alpha = 0.15$ on the measurement deltas, minimizing the possibility of false alarms. For noise level of $\alpha = 0.15$, the FLS shows degradation in damage detection as the overlaps in the damage evaluation parameter increases and it becomes very difficult to classify the damage. But still, the proposed FLS gives very good accuracy in classifying "Slight Damage at Tip", which is the most difficult damage to detect in a cantilever beam structure.

Typically, model-based damage detection algorithms are developed using a good validated baseline model of the proposed structure, for example, an aircraft wing or helicopter rotor blade. The damaged conditions of the structure are simulated using the mathematical model. The damage detection algorithm is then deployed across a fleet of structures. In such cases, variability exists among the structures due to random uncertainty in addition to the measurement uncertainty due to sensors. The proposed approach in this paper results in a robust damage detection system which is less sensitive to both measurement and model uncertainty. By connecting probabilistic MCS to the development of Gaussian fuzzy systems, we also connect the disparate areas of probability and fuzzy logic in a novel manner to address uncertainty issues in damage detection.

Table 9
Success rate in percent for the FLS with sliding window defuzzifier at different noise levels (α) for single damage case with different geometric uncertainty (β) in beam taper angle.

Rule no.	COV of geometric uncertainty (β)					
	$\beta = 0.5$ percent		$\beta = 1.0$ percent		$\beta = 1.5$ percent	
	$\alpha = 0.10$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.15$
Undamaged	99.83	99.59	99.68	99.23	99.47	99.08
Slight Damage at Root	98.12	92.13	94.25	88.34	85.25	82.13
Moderate Damage at Root	96.45	91.53	93.63	88.10	89.24	83.24
Severe Damage at Root	99.21	95.67	97.41	90.27	92.33	86.57
Slight Damage at Inboard	93.46	86.37	87.61	80.61	81.63	78.59
Moderate Damage at Inboard	100	94.86	96.35	90.24	93.26	86.63
Severe Damage at Inboard	100	94.98	98.32	90.86	93.11	84.23
Slight Damage at Center	96.72	88.63	92.37	86.43	81.27	76.14
Moderate Damage at Center	100	98.18	95.92	90.23	92.17	85.73
Severe Damage at Center	97.94	92.96	94.63	85.81	87.07	81.38
Slight Damage at Outboard	97.31	86.38	86.31	80.57	79.30	72.94
Moderate Damage at Outboard	99.01	94.72	96.58	86.37	89.61	83.41
Severe Damage at Outboard	99.76	95.53	98.46	89.52	95.35	84.18
Slight Damage at Tip	98.17	96.16	94.20	86.56	87.12	81.34
Moderate Damage at Tip	95.43	93.41	93.73	86.64	85.15	81.13
Severe Damage at Tip	99.51	97.14	94.52	85.21	89.11	81.01
Average S_R	98.18	93.64	94.62	87.81	88.78	82.98

Table 10
Success rate in percent for the FLS with sliding window defuzzifier at different noise levels (α) for multiple damage case with different geometric uncertainty (β) in beam taper angle.

Rule no.	COV of geometric uncertainty (β)					
	$\beta = 0.5$ percent		$\beta = 1.0$ percent		$\beta = 1.5$ percent	
	$\alpha = 0.10$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.15$	$\alpha = 0.10$	$\alpha = 0.15$
Undamaged	99.83	99.59	99.68	99.23	99.47	99.08
Slight Damage at $\text{Root}_{\text{multiple}}$	95.21	90.37	93.14	86.37	86.27	81.82
Moderate damage at $\text{Root}_{\text{multiple}}$	98.23	92.20	95.43	86.67	88.13	83.10
Severe damage at $\text{Root}_{\text{multiple}}$	99.17	95.86	96.21	88.32	90.19	82.72
Slight Damage at $\text{Inboard}_{\text{multiple}}$	94.72	87.16	89.64	82.85	85.34	78.63
Moderate damage at $\text{Inboard}_{\text{multiple}}$	100	95.27	98.20	90.30	94.01	85.89
Severe damage at $\text{Inboard}_{\text{multiple}}$	97.62	91.14	92.13	84.23	86.76	81.37
Slight Damage at $\text{Center}_{\text{multiple}}$	95.17	90.63	90.47	85.21	87.17	82.75
Moderate damage at $\text{Center}_{\text{multiple}}$	99.05	96.47	97.30	90.63	91.61	87.12
Severe damage at $\text{Center}_{\text{multiple}}$	96.35	91.52	93.76	86.66	87.87	83.60
Slight Damage at $\text{Outboard}_{\text{multiple}}$	93.83	87.37	88.35	80.74	84.19	76.81
Moderate damage at $\text{Outboard}_{\text{multiple}}$	96.21	92.17	93.19	85.52	97.46	81.67
Severe damage at $\text{Outboard}_{\text{multiple}}$	98.07	92.10	95.38	88.26	89.27	82.65
Slight Damage at $\text{Tip}_{\text{multiple}}$	94.93	91.84	90.61	84.73	86.07	80.77
Moderate damage at $\text{Tip}_{\text{multiple}}$	97.54	94.89	93.27	86.30	87.74	81.38
Severe damage at $\text{Tip}_{\text{multiple}}$	99.73	96.66	93.60	83.56	88.83	79.80
Average S_R	97.23	93.01	93.77	86.85	89.40	83.07

8.3. Closing remarks

Since the primary aim of the paper is to demonstrate a new damage detection algorithm, the results in this work were illustrated for a cantilever beam using curvature-based damage indicator. There are some limitations of the damage indicator and structure selected which need to be pointed out.

The success of the FLS depends on the knowledge base available for damage identification. Thus, the FLS cannot detect a damage which does not have a rule representation available with the FLS. It can possibly detect a damage having closest features (damage indicators) to that of the existing damage condition. It is possible that we loose uniqueness of the rules

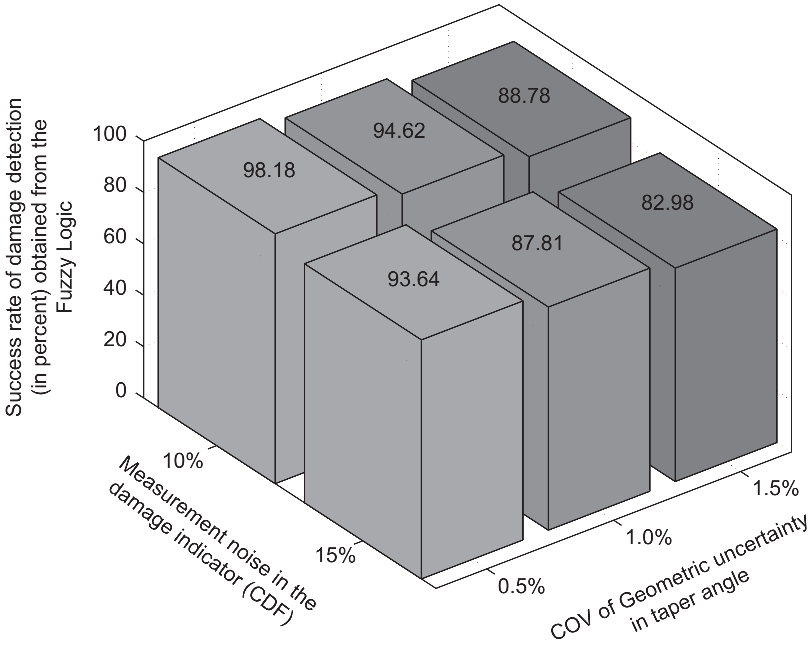

Fig. 9. Plot for success rate of the fuzzy logic system at different geometric uncertainty and noise levels for "single damage case".

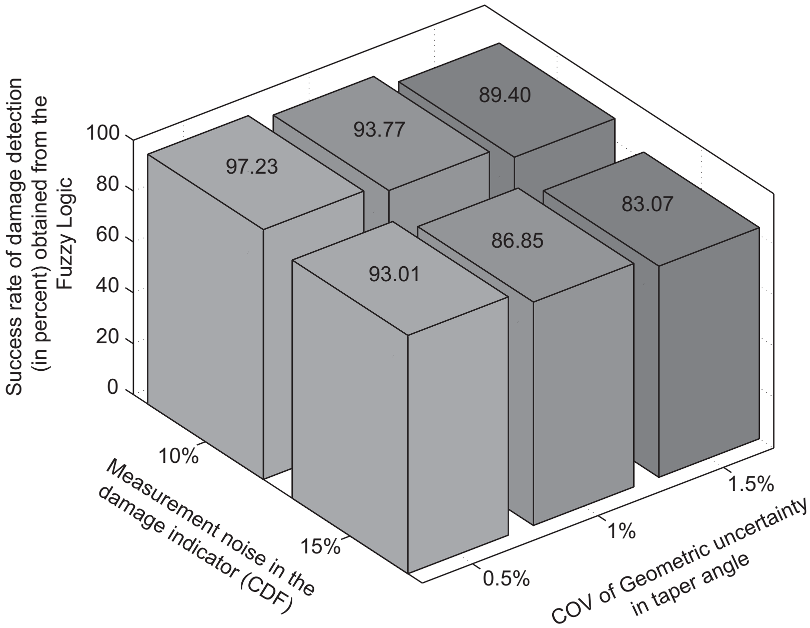

Fig. 10. Plot for success rate of the fuzzy logic system at different geometric uncertainty and noise levels for "multiple damage case".

for two different damage states for a complex structure (or possibly for damages occurring at more refined levels). Then we would need more powerful damage indicators based on wavelet or HHT transforms [49,50], which are more sensitive and localized with respect to damage to maintain the uniqueness of the fuzzy rules. These issues are subjects for future research. Though using numerical simulations with added noise is an established approach for developing and testing damage detection algorithms [11,35], experimental validation is the true test. Again, that is a subject for future research.

9. Conclusions

A fuzzy logic system with a new fault isolation (sliding window) technique is developed for damage detection in structures using the curvature damage factor vector. The CDF vector calculated from changes in the mode shape curvature vectors corresponding to the first six modes of the beam structure is used as a representative damage indicator. Test results are obtained to simulate the uncertainties in geometric properties of the beam as well as noise in the measurement deltas. The following conclusions are made from this study:

1. There are large overlaps in the damage evaluation parameter for different damage levels due to randomness in the geometric properties itself, which increases with the noise contamination. These overlaps cause difficulties in damage evaluation.
2. CDF clearly indicates the location of existing damage in single as well as multiple damage situations in the presence of uncertainty. This also reduces the conditions for false alarms as the existence of damage is clearly detectable. But quantification of the damage is very difficult in the presence of geometric and measurement uncertainties as the damage indicator (CDF) shows large overlaps for different damage levels.
3. The proposed FLS with newly proposed sliding window fault isolation technique detects damage with an average success rate of 94.62 percent for single damage case and 93.77 percent for multiple damage case. These results are for an added noise level $\alpha = 0.10$ on the measurement deltas.
4. The FLS degrades in detecting damages for increasing noise levels as the overlaps in the damage evaluation parameter increase considerably, creating difficulties for damage assessment.
5. The new FLS tested with different levels of measurement noise classifies damages with an average accuracy of 87.73 percent and 86.94 percent for added noise level $\alpha = 0.15$ in single and multiple damage conditions, respectively. It also classifies the undamaged structure with an accuracy of 99.23 percent for added noise level up to $\alpha = 0.15$ on the measurement deltas, avoiding the possibility of false alarms.
6. Even a slight damage at the tip location is correctly classified by the FLS.

Based on the numerical results in this work, the fuzzy system based on curvature damage factor is proposed as a robust tool for structural damage detection.

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References

[1] D. Montalvao, N.M.M. Maia, A.M.R. Ribeiro, A review of vibration-based structural health monitoring with special emphasis on composite materials, *Shock and Vibration Digest* 38 (4) (2006) 295–324.
[2] E.P. Carden, P. Fanning, Vibration based condition monitoring: a review, *Structural Health Monitoring* 3 (4) (2004) 355–377.
[3] C.P. Ratcliffe, A frequency and curvature based experimental method for locating damage in structures, *Journal of Vibration and Acoustics—Transactions of the ASME* 122 (3) (2000) 324–329.
[4] S.W. Doebling, C.R. Farrar, M.B. Prime, A summary review of vibration based damage identification methods, *Shock and Vibration Digest* 30 (3) (1998) 91–105.
[5] O.S. Salawu, Detection of structural damage through changes in frequency: a review, *Engineering Structures* 19 (9) (1997) 718–723.
[6] J.T. Kim, Y.S. Ryu, H.M. Cho, N. Stubbs, Damage identification in beam-type structures: frequency-based method vs mode-shape-based method, *Engineering Structures* 25 (1) (2003) 57–67.
[7] N. Stubbs, J.T. Kim, Damage localization in structures without baseline modal parameters, *AIAA Journal* 34 (8) (1996) 1649–1654.
[8] J. Chen, J.A. Garba, On-orbit damage assessment for large space structures, *AIAA Journal* 26 (9) (1988) 1119–1126.
[9] A.K. Pandey, M. Biswas, Damage detection in structures using changes in flexibility, *Journal of Sound and Vibration* 169 (1) (1994) 3–17.
[10] C.R. Farrar, D.A. Jauregui, Comparative study of damage identification algorithm applied to a bridge: I. Experiment, *Smart Materials & Structures* 7 (5) (1998) 704–719.
[11] A.K. Pandey, M. Biswas, M.M. Samman, Damage detection from changes in curvature mode shapes, *Journal of Sound and Vibration* 145 (2) (1991) 321–332.
[12] M. Sahin, R.A. Shenoi, Quantification and localisation of damage in beam-like structures by using artificial neural networks with experimental validation, *Engineering Structures* 25 (2003) 1785–1802.
[13] C.P. Ratcliffe, W.J. Bagaria, Vibration technique for locating delamination in a composite beam, *AIAA Journal* 36 (6) (1998) 1074–1077.
[14] C.S. Hamey, W. Lestari, P. Qiao, G. Song, Experimental damage identification of carbon/epoxy composite beams using curvature mode shapes, *Structural Health Monitoring* 3 (4) (2004) 333–353.
[15] P.H. Qiao, K. Lu, W. Lestari, J. Wang, Curvature mode shape-based damage detection in composite laminated plates, *Composite Structures* 80 (3) (2007) 409–428.
[16] M.M.A. Wahab, G.D. Roeck, Damage detection in bridges using modal curvatures: application to a real damage scenario, *Journal of Sound and Vibration* 226 (2) (1999) 217–235.
[17] A. Datta, S. Talukdar, Damage detection in bridges using accurate modal parameters, *Finite Elements in Analysis and Design* 40 (3) (2004) 287–304.
[18] H. Guo, L. Zhang, A weighted balance evidence theory for structural multiple damage localization, *Computer Methods in Applied Mechanics and Engineering* 195 (44–47) (2006) 6225–6238.
[19] A. O'Hagan, J.E. Oakley, Probability is perfect but we can't elicit it perfectly, *Reliability Engineering & System Safety* 85 (1–3) (2004) 239–248.

[20] Y. Xia, H. Hao, J.M.W. Brownjohn, P.Q. Xia, Damage identification of structures with uncertain frequency and mode shape data, *Earthquake Engineering & Structural Dynamics* 31 (5) (2002) 1053–1066.
[21] J.D. Collins, G.C. Hart, T.K. Hasselman, K. Kennedt, System identification of structures, *AIAA Journal* 12 (2) (1974) 185–190.
[22] W. Gao, Natural frequency and mode shape analysis of structures with uncertainty, *Mechanical Systems and Signal Processing* 21 (1) (2007) 24–39.
[23] J. Cheng, R.C. Xiao, Probabilistic free vibration analysis of beams subjected to axial loads, *Advances in Engineering Software* 38 (1) (2007) 31–38.
[24] R.R. Melchers, *Structural Reliability Analysis and Prediction*, Wiley, New York, 1999.
[25] A. Haldar, S. Mahadevan, *Reliability Assessment Using Stochastic Finite Element Analysis*, Wiley, New York, 2000.
[26] E. Hofer, M. Kloos, B. Krzykacz-Hausmanna, J. Peschkea, M. Woltereckb, An approximate epistemic uncertainty analysis approach in the presence of epistemic and aleatory uncertainties, *Reliability Engineering & System Safety* 77 (3) (2002) 229–238.
[27] P.L. Liu, Identification and damage detection of trusses using modal data, *Journal of Structural Engineering* 121 (4) (1995) 599–608.
[28] J.P. Sawyer, S.S. Rao, Structural damage detection and identification using fuzzy logic, *AIAA Journal* 38 (12) (2000) 2328–2335.
[29] P.M. Pawar, R. Ganguli, Genetic fuzzy system for damage detection in beams and helicopter rotor blades, *Computer Methods in Applied Mechanics and Engineering* 192 (16–18) (2003) 2031–2057.
[30] M. Chandrashekhar, R. Ganguli, Uncertainty handling in structural damage detection using fuzzy logic and probabilistic simulation, *Mechanical Systems and Signal Processing* 23 (2) (2009) 384–404.
[31] X. Yong, H. Hong, Statistical damage identification of structures with frequency changes, *Journal of Sound and Vibration* 263 (4) (2003) 853–870.
[32] N. Bakhary, H. Hao, A.J. Deeks, Damage detection using artificial neural network with consideration of uncertainties, *Engineering Structures* 29 (11) (2007) 2806–2815.
[33] S. Chinchalkar, D.L. Taylor, Geometric uncertainties in finite element analysis, *Computing Systems in Engineering* 5 (2) (1994) 159–170.
[34] L. Zadeh, Fuzzy logic = computing with words, *IEEE Transactions on Fuzzy Systems* 4 (2) (1996) 103–111.
[35] R. Ganguli, A fuzzy logic system for ground based structural health monitoring of a helicopter rotor using modal data, *Journal of Intelligent Material Systems and Structures* 12 (6) (2001) 397–407.
[36] F. Aymerich, S. Meili, Ultrasonic evaluation of matrix damage in impacted composite laminates, *Composites: Part B* 31 (1) (2000) 1–6.
[37] J.N. Reddy, *An Introduction to the Finite Element Method*, McGraw-Hill Inc., New York, 2005.
[38] Z. Qiu, X. Wang, M.I. Friswell, Eigenvalue bounds of structures with uncertain-but-bounded parameters, *Journal of Sound and Vibration* 282 (2005) 297–312.
[39] B.N. Singh, D. Yadav, N.G.R. Iyengar, Natural frequencies of composite plates with random material properties using higher-order shear deformation theory, *International Journal of Mechanical Sciences* 43 (10) (2001) 2193–2214.
[40] G.V. Vinckenroy, W.P.D. Wilde, The use of Monte Carlo techniques in statistical finite element methods for the determination of structural behavior of composite materials structural components, *Composite Structures* 32 (1–4) (1995) 247–253.
[41] J.D. Kaplunov, E.V. Nolde, B.F. Shorr, A perturbation approach for evaluating natural frequencies of moderately thick elliptic plates, *Journal of Sound and Vibration* 281 (3–5) (2005) 905–919.
[42] L. Kong, R.G. Parker, Approximate eigensolutions of axially moving beams with small flexural stiffness, *Journal of Sound and Vibration* 276 (1–2) (2004) 459–469.
[43] V. Papadopoulos, M. Papadrakakis, G. Deodatis, Analysis of mean and mean square response of general linear stochastic finite element systems, *Computer Methods in Applied Mechanics and Engineering* 195 (41–43) (2006) 5454–5471.
[44] G. Stefanou, M. Papadrakakis, Stochastic finite element analysis of shells with combined random material and geometric properties, *Computer Methods in Applied Mechanics and Engineering* 193 (1–2) (2004) 139–160.
[45] T.J. Ross, *Fuzzy Logic with Engineering Applications*, McGraw-Hill Inc., Singapore, 1997.
[46] Y. Jin, W.V. Seelen, B. Sendhoff, B., Extracting interpretable fuzzy rules from RBF neural networks, Report, Ruhr-Universitat Bochum Institut fur Neuroinformatik, 44780 Bochum, FRG, 2000, IR-INI 2000-02, ISSN 0943-2752.
[47] S. Abe, M.S. Lin, A method for fuzzy rules extraction directly from numerical data and its application to pattern recognition, *IEEE Transactions on Fuzzy Systems* 3 (1) (1995) 18–28.
[48] Z. Chi, H. Yan, T. Pham, *Fuzzy Algorithms: with Applications to Image Processing and Pattern Recognition*, World Scientific, Singapore, 1998.
[49] J. Grabowskaa, M. Palaczb, M. Krawczuka, Damage identification by wavelet analysis, *Mechanical Systems and Signal Processing* 22 (7) (2008) 1623–1635.
[50] H.G. Chen, Y.J. Yan, W.H. Chen, J.S. Jiang, L. Yu, Z.Y. Wu, Early damage detection in composite wingbox structures using Hilbert–Huang transform and genetic algorithm, *Structural Health Monitoring* 6 (4) (2007) 281–297.